Let's Talk

Promoting Mathematical Discourse in the Classroom

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As part of reform-based mathematics, much discussion and research has focused on the idea that mathematics should be taught in a way that mirrors the nature of the discipline (Lampert 1990)—that is, have students use mathematical discourse to make conjectures, talk, question, and agree or disagree about problems in order to discover important mathematical concepts. In fact, communication, of which student discourse is a part, is so important that it is one of the Standards set forth in *Principles and Standards for School Mathematics* (NCTM 2000).

The use of discourse in the mathematics classroom, however, can be difficult to implement and manage. The same students participate in every discussion while others contribute only when called on, and even then their contributions are sparse. Some students make comments that relate to procedure but never reach the deeper-level mathematical concepts. This article discusses what research tells us about mathematics discourse in the classroom and explores the ways in which teachers establish the classroom community at the beginning of the year, facilitate discussion, and assess the quality of discourse.
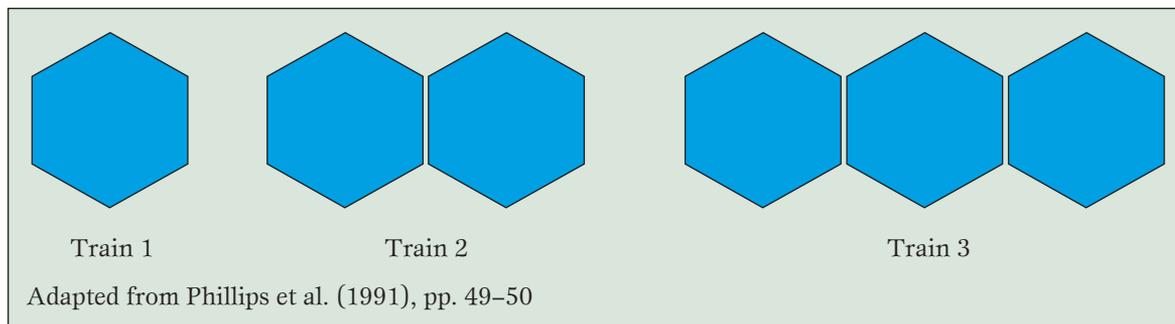
Train 1   Train 2   Train 3

Adapted from Phillips et al. (1991), pp. 49–50

Fig. 1 Hexagon perimeter train

SETTING THE STAGE

How do I set up the classroom community to encourage students to participate?

Teachers send messages about what is important to them by the way they establish their classroom community. Of course, accuracy is essential in mathematics, but to encourage discourse, teachers must show students that they value understanding concepts rather than just getting the right answer. Turner et al. (2003) grouped the messages that teachers send into four categories: (1) messages about tasks, learning, and expectations for students; (2) relationships with the teacher; (3) relationships among students; and (4) rules and management structures. Teachers in classrooms supportive of discourse showed enthusiasm for learning, set expectations that all students would learn, and established classroom relationships and management systems based on respect.

One could rightly argue that these principles for establishing a community are true of classrooms in general and are not specific to mathematics classrooms and mathematics classroom discourse. Yackel and Cobb (1996), however, argue that establishing a mathematical community also includes sociomathematical norms, the norms of the mathematics community. Although these norms may never be overtly stated, through discussion the teacher and students come to an understanding about what counts as mathematical difference, sophistication, and explanation. Consider two students' responses to a task that asks them to find a rule for determining the perimeter of any given hexagon train (see **fig. 1**).

Student 1

Solution: $p = 4(n - 2) + 10$
Explanation: The middle blocks of the train have four sides out of six total sides that can be counted in the perimeter. So n equals the number of blocks. I took away the two blocks on the end since I'm only counting the middle. Then I multiplied by four to find the number of sides that can be counted for the perimeter. The two end blocks each have five sides showing, so I added ten.

Student 2

Solution: $p = 4n + 2$
Explanation: Each hexagon has at least four sides on the outside of the train, so I multiplied four by the number of hexagons (n). The hexagons on the end have one extra side, so I added two for the two sides on the end.

Are the two solutions different mathematically? Are the solutions efficient? Are the explanations provided acceptable? The answers to these questions will be negotiated as the classroom community participates in discourse, but they will ultimately depend on the teacher. Teachers send both explicit and hidden messages about what they value in mathematics and what they expect of students.

FACILITATING DISCOURSE

My students understand the expectations and norms. Now what do I do?

There is a misconception that the shift toward the use of classroom discourse in teaching mathematics means that the teacher simply presents the problem and then stands aside while students discuss and solve it (Chazan and Ball 1995). The teacher's instructional role is perceived as "don't tell the answer." This perception severely underrates the complexity of the teacher's role in classroom discourse (Chazan and Ball 1995). So what should teachers do during discussions to increase participation and conceptual understanding? There are two aspects of teacher discourse to be considered: cognitive discourse and motivational discourse.

Cognitive discourse refers to what the teacher says to promote conceptual understanding of the mathematics itself. Kazemi and Stipek (1997) found that some inquiry-based classrooms, described as *low-press*, are still not effective in facilitating student discourse because they focus only on explanations of procedure and do not link to a conceptual understanding of mathematics. In the following example, a teacher and a student are discussing the student's solution to the Skeleton Tower problem (see **fig. 2**).
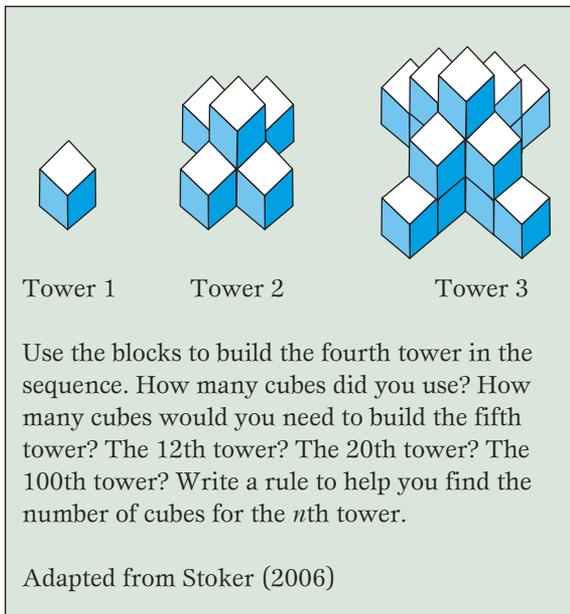
Use the blocks to build the fourth tower in the sequence. How many cubes did you use? How many cubes would you need to build the fifth tower? The 12th tower? The 20th tower? The 100th tower? Write a rule to help you find the number of cubes for the nth tower.

Adapted from Stoker (2006)

Fig. 2 Skeleton Tower problem

Ms. D. Please explain how you found the rule for the towers.

S. The center of each tower has the same number of cubes as the tower number, so that equals n cubes.

Ms. D. Okay, then what?

S. There are four arms coming out from the center in the shape of triangles.

Ms. D. Triangles?

S. Yeah, when you flip them over you get two rectangles. The height of the rectangle is the same as the center, and the width is one less. So $2n(n - 1) + n$ gives you the number of cubes.

Ms. D. $2n(n - 1) + n$. Does everyone agree? [*"Yeahs" heard from around the room.*] Does everyone understand how he got the answer? [*More "yeahs" from the class.*] Okay, who else has a solution?

In contrast, in *high-press* classrooms, teachers push students to link the strategies and procedures used to the underlying concepts. The following exchange begins in the same way as the previous one. In this example, however, the teacher presses the student for more information about his thinking.

S. The center of each tower has the same number of cubes as the tower number, so that equals n cubes.

Ms. K. Okay, then what?

S. There are four arms coming out from the center in the shape of triangles.

Ms. K. Can you explain what you mean by triangles?

S. The cubes look like the shape of a triangle.

Ms. K. Let's be sure everyone understands. Can you show us one of the triangles on the model you built of the fourth tower?

S. Sure. When you look at one of the arms coming

out from the center [*pulls the cubes away from the rest of the model*], you have a piece with three cubes on the bottom, two on the middle level, and one on the top level. It looks like a triangle.

Ms. K. Okay, I see. Why are the triangles important?

S. Because if I can figure out how many cubes are in the triangles for each tower, I can add that number to the center tower and figure out how many cubes total. [*The exchange continues as the student continues explaining.*]

In addition to helping students make connections, teachers of high-press classrooms take better advantage of helping students learn from mistakes and stress individual accountability so that all students are engaged.

The issue of engagement necessitates the second type of teacher discourse, *motivational discourse*. Motivational discourse refers not only to praise offered to students but also to supportive and non-supportive statements teachers make that encourage or discourage participation in mathematics classroom discussions. Students' lack of participation in classroom discourse can be a result of self-handicapping, failure avoidance, or a preference for avoiding novelty (Turner et al. 2002). Sometimes students who disagree remain silent rather than express a mathematical argument (Lampert 1990). Turner et al. (2002) found that when teachers used supportive motivational discourse in addition to pressing for conceptual understanding, the reported levels of these behaviors decreased.

Supportive motivational discourse occurs when teachers focus on learning through mistakes, collaboration, persistence, and positive affect (Turner et al. 2003). Consider the following exchange in which a student explains her solution to the teacher.

Ms. K. Explain to the class how you built the fourth tower.

Susan. It doesn't look like the picture.

Ms. K. If you explain how you thought about it, maybe we can help you figure out where you're making a mistake. I see some other towers around the room that don't look like the picture. As you think aloud, maybe together we can figure out how to build it.

Though this is a brief exchange, the messages sent by the teacher are clear. Mistakes are an opportunity for learning, and the learning is a collaborative process in which all students are expected to participate. Conversely, nonsupportive motivational discourse occurs when teachers emphasize getting the right answers without mistakes, compare or highlight individual successes or failures, or use sarcasm or humiliation (Turner et al. 2003).

Table 1	
Levels of Discourse in a Mathematics Classroom	
Levels	**Characteristics of Discourse**
0	The teacher asks questions and affirms the accuracy of answers or introduces and explains mathematical ideas. Students listen and give short answers to the teacher's questions.
1	The teacher asks students direct questions about their thinking while other students listen. The teacher explains student strategies, filling in any gaps before continuing to present mathematical ideas. The teacher may ask one student to help another by showing how to do a problem.
2	The teacher asks open-ended questions to elicit student thinking and asks students to comment on one another's work. Students answer the questions posed to them and voluntarily provide additional information about their thinking.
3	The teacher facilitates the discussion by encouraging students to ask questions of one another to clarify ideas. Ideas from the community build on one another as students thoroughly explain their thinking and listen to the explanations of others.

Adapted from Hufferd-Ackles, Fuson, and Sherin (2004)

The following encounter provides an example of nonsupportive motivational discourse in which the teacher is more concerned with the right answer than with the student's thinking about the task.

Ms. D. Explain how you built the fourth tower.
Bill. [*Holds up the tower he built with his partner.*]
Ms. D. This is not the fourth tower in the pattern. Does it look like it should be? You should be able to build it with a picture. The first level has 1 cube; the second level has 5; the third level has 9. It's going up by 4 each time. So, how many cubes in the fourth level?
Bill. Thirteen.
Ms. D. Nine plus four equals thirteen. Now build it like the picture. Who thinks they have it right?

As these examples indicate, the teacher's role in discourse is complex. Teachers must be conscious about the statements they make and the questions they ask so that all students are encouraged to participate.

ASSESSING DISCOURSE

How do I know if the discourse in my classroom is successful?

Mathematics discourse does not happen overnight, particularly if students have experienced only teacher-directed, procedure-oriented mathematics classrooms. As a result, mathematics classroom discourse is a dynamic process that is often hard to assess. Hufferd-Ackles, Fuson, and Sherin (2004) created a framework to describe and evaluate the process a class goes through when discourse is introduced. Four categories are examined—questioning, explanation of mathematical thinking,

source of mathematical ideas, and responsibility for learning. A scale of 0 to 3 is used, where level 0 refers to a traditional, teacher-directed class, and level 3 is reached when the teacher participates as a member of the community and assists only as needed (see **table 1**). Although this framework serves as a good indicator for assessing the discourse level of the whole class, it does not assess individual students. Teachers need to be aware of how individual students are participating so that they can encourage and scaffold students who are not participating in the discourse.

CONCLUSION

Participating in a mathematical community through discourse is as much a part of learning mathematics as the conceptual understanding of the mathematics itself. As students learn to make and test conjectures, question, and agree or disagree about problems, they are learning the essence of what it means to do mathematics. If all students are to be engaged, teachers must foster classroom discourse by providing a welcoming community, establishing norms, using supportive motivational discourse, and pressing for conceptual understanding. As Johnston (2004) puts it, "In other words, the language that teachers (and their students) use in classrooms is a big deal" (p. 10).

REFERENCES

Chazan, D., and D. Ball. "Beyond Exhortations Not to Tell: The Teacher's Role in Discussion-Intensive Mathematics Classes." (Craft Paper 95-2). East Lansing, MI: National Center for Research on Teacher Learning, 1995.

Hufferd-Ackles, K., K. Fuson, and M. Sherin. "Describing Levels and Components of a Math-

Talk Learning Community." *Journal for Research in Mathematics Education* 35 (March 2004): 81–116.

Johnston, P. *Choice Words: How Our Language Affects Children's Learning.* Portland, ME: Stenhouse Publishers, 2004.

Kazemi, E., and D. Stipek. "Pressing Students to Be Thoughtful: Promoting Conceptual Thinking in Mathematics." Paper presented at the annual meeting of the American Educational Research Association, Chicago, 1997.

Lampert, M. "When the Problem Is Not the Question and the Solution Is Not the Answer: Mathematical Knowing and Teaching." *American Education Research Journal* 27 (Spring 1990): 29–63.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, VA: NCTM, 2000.

Phillips, E., with T. Gardella, C. Kelly, and J. Stewart. *Patterns and Functions:* Addenda Series, Grades 5–8. Reston, VA: National Council of Teachers of Mathematics, 1991.

Stoker, J. "Promoting the Professional Development of Mathematics Teachers through Aligned Assessment Tasks." 2006. www.aare.edu.au/01pap/sto01354.htm.

Turner, J. C., D. K. Meyer, C. Midgley, and H. Patrick. "Teacher Discourse and Sixth Graders' Reported Affect and Achievement Behaviors in Two High-Mastery/High-Performance Mathematics Classrooms." *Elementary School Journal* 103 (March 2003): 357–82.

Turner, J., C. Midgley, D. Meyer, M. Gheen, E. Anderman, Y. Kang, and H. Patrick. "The Classroom Environment and Students' Reports of Avoidance Strategies in Mathematics: A Multimethod Study." *Journal of Educational Psychology* 94 (March 2002): 88–106.

Yackel, E., and P. Cobb. "Sociomathematical Norms, Argumentation, and Autonomy in Mathematics." *Journal for Research in Mathematics Education* 27 (July 1996): 458–77. ∞

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